PDE

Mini Workshop on PDEs

26 E 27 DE FEVEREIRO DE 2026

UNIVERSIDADE ESTADUAL DE LONDRINA

Anais do MWPDE

Book of Abstracts

REALIZAÇÃO E APOIO FINANCEIRO

pós-graduação pgmac Matemática Aplicada Computacional

proex Pró-Reitoria de Extensão, Cultura e Sociedade

Universidade Estadual de Londrina

ARAUCARIA

PARANÁ GOVERNO DO ESTADO SECRETARIA DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR

REALIZAÇÃO

FOMENTO

Mini Workshop on PDEs (MWPDE 2026)
26 e 27 de Fevereiro de 2026
Universidade Estadual de Londrina

Londrina – Paraná – Brasil

LOCAL DAS ATIVIDADES

As atividades do MWPDE serão realizadas no LABESC – Laboratório Escola de Pós-Graduação (Sala 04) da Universidade Estadual de Londrina.

MAPA

Figura 1: Mapa do campus da UEL.

Acesse o mapa em:
https://maps.app.goo.gl/ueZfcfgQR9yS3hNY8

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Ma To Fu – UnB/Brasil

Marcelo Moreira Cavalcanti – UEM/Brasil

Paolo Piccione – USP/Brasil

Valéria Neves Domingos Cavalcanti – UEM/Brasil

Programação

MWPDE 2026

O MWPDE contará com sessões de Palestras (PA) e Pôsteres (PO) conforme a programação abaixo.

Horário	26/02 (QUI)	27/02 (SEX)
08:00 - 08:30	Recepção	--
08:30 - 09:00	Abertura	Pedro R. Lima (UFRRJ)
09:00 - 09:30	Ma To Fu (UnB)	Angelo Guimarães (UEG)
09:30 - 10:00		Mirelson M. Freitas (UnB)
10:00 - 10:30	Coffee Break	
10:30 - 11:00	Eduardo H. G. Tavares (DGUT)	Estefani M. Moreira (IME-USP)
11:00 – 11:30	Bianca M. R. Calsavara (UNICAMP)	Leonel G. Delatorre (UFSM)
11:30 – 13:30	Almoço	
13:30 – 14:00	Anderson J. A. Ramos (UFPA)	Maykel B. Belluzi (UFSCar)
14:00 – 14:30	Monisse P. Alves (UTFPR)	Wellington J. Corrêa (UTFPR)
14:30 – 15:00	Heraclio L. L. Lázaro (ICMC-USP)	Rodrigo N. Monteiro (UEL)
15:00 – 15:15	Coffee Break	
15:15 – 15:55	Pôster (PO) – Sessão 1 PO1, PO2, PO3, PO4	Pôster (PO) – Sessão 2 PO5, PO6, PO7
16:00 – 16:30	Gabriel E. B. Moraes (UEL)	Carlos R. Takaessu Jr. (UFSCar)
16:30 – 17:00	Vando Narciso (UEMS)	Alberto S. Noé (UFAM)
17:00 – 17:10	--	Encerramento
19:30 – 11:00	Jantar de confraternização	

Relação dos Palestrantes e Sessões das Palestras

26/02

09:00 – 10:00 - Ma To Fu (UnB)
PA1. Mathematical modeling for the analysis of PDEs

10:30 – 11:00 - Eduardo H. G. Tavares (DGUT)
PA2. Principles of basic research

11:00 – 11:30 - Bianca M. R. Calsavara (UNICAMP)
PA3. Characterization of the uniform exponential stability for a thermo-viscoelastic Timoshenko system with memory

13:30 – 14:00 - Anderson J. A. Ramos (UFPA)
PA4. Structural and microthermal effects on the exponential stabilization of Poroelastic systems

14:00 – 14:30 - Monisse P. Alves (UTFPR)
PA5. Navier-Stokes Equations on Thin Domains

14:30 – 15:00 - Heraclio L. L. Lázaro (ICMC-USP)
PA6. Non-autonomous Dynamical Systems with Applications to Non-cylindrical Problems in PDEs

16:00 – 16:30 - Gabriel E. B. Moraes (UEL)
PA7. On the stability of Bresse systems: observability inequality via resolvent equation

16:30 – 17:00 - Vando Narciso (UEMS)
PA8. Attractors for the Wave Equation With Critical Growth Nonlinearities

27/02

08:30 – 09:00 - Pedro R. Lima (UFRRJ)
PA9. On the stabilization of twisted Bresse systems

09:00 – 09:30 - Angelo Guimarães (UEG)
PA10. Existence and multiplicity of solution to fourth-order equations with critical Sobolev growth

09:30 – 10:00 - Mirelson M. Freitas (UnB)
PA11. Random Quasi-Stability and Long-Time Dynamics of Stochastic Piezoelectric Systems with Magnetic and Thermal Effects

10:30 – 11:00 - Estefani M. Moreira (IME-USP)
PA12. Existence of an inertial manifold for a nonlocal quasilinear scalar one-dimensional problem

11:00 – 11:30 - Leonel G. Delatorre (UFSM)
PA13. Timoshenko-Boltzmann systems with non-smooth memory kernels

13:30 – 14:00 - Maykel B. Belluzi (UFSCar)
PA14. Semilinear Evolution Equations with time-dependent linear operators: existence of pullback attractors and permanence under perturbations

14:00 – 14:30 - Wellington J. Corrêa (UTFPR)
PA15. Asymptotic Behavior of Nonlinear Quintic Defocusing KGS with a Localized Damping

14:30 – 15:00 - Rodrigo N. Monteiro (UEL)
PA16. Lack of exponential stability for fully dissipative Timoshenko systems

16:00 – 16:30 - Carlos R. Takaessu Jr. (UFSCar)
PA17. Extension of the shadowing property to infinite dimensional dynamical systems

16:30 – 17:00 - Alberto S. Noé (UFAM)
PA18. Exact analytical solution of the parabolic two-step model for nanoscale heat conduction

Relação das Apresentações e Sessões dos Pôsteres

26/02

15:15 – 15:25 - Beatriz Signori Lonardoni (UEM)
PO1. On the orbital stability of periodic snoidal-type solutions to the ϕ^4-equation

15:25 – 15:35 - Leonardo G. Ronchin Alves (UEM)
PO2. Comparison of long-time dynamics between the Bresse System and the Timoshenko-wave system

15:35 – 15:45 - Isabela Caroline de Paula (UEL)
PO3. Aplicação de Equações Diferenciais Parciais no Realce de Imagens de Impressões Digitais

15:45 – 15:55 - Rafael Gabardo da Camara (UEL)
PO4: Asymptotic dynamics of an extensible beam model with fractional nonlocal damping

27/02

15:15 – 15:25 - Paulo F. Mercadante Damazio (UEM)
PO5: Attractors for a two-dimensional Lamé system with exponential nonlinearity

15:25 – 15:35 - Nathalia Alvares (UEL)
PO6: Comparison of the numerical accuracy of triangular and quadrangular meshes in the finite volume method

15:35 – 15:45 - Matheus Sadao Susuki (UEL)
PO7: Control Systems on Lie Groups

Book of Abstracts

MWPDE 2026

Conteúdo

Exact analytical solution of the parabolic two-step model for nanoscale heat conduction

Alberto S. N. Chairuca
Universidade Federal do Amazonas, Brasil
(joint work with **A. J. A. Ramos, J. A. R. Nascimento and Y. S. Gonçalves**)

This work presents an analytical solution to the parabolic two-step model, widely used in modeling nanoscale heat conduction, especially in metallic materials subjected to ultrashort laser pulses. The model describes the thermal coupling between the electron gas and the crystal lattice through a system of coupled differential equations. The main contribution, is the derivation of an explicit solution using the technique of separation of variables and Fourier series, without the need to assume the null initial velocity (**NIV**) condition, which is often imposed in the literature. The analytical solution provides a rigorous description of the transient behavior of the temperatures in the electronic and lattice subsystems, revealing the presence of exponential decay modes. Additionally, computational simulations are carried out to illustrate the rapid thermal dissipation in the electronic subsystem, followed by a slower redistribution in the crystal lattice, behavior that is characteristic of electron-phonon coupling at the nanoscale.

Problem

This seminar addresses the problem modeled by:

$$\begin{cases} \dfrac{\partial T^e}{\partial t} - \dfrac{\mathrm{Kn}^2}{3}\dfrac{\partial^2 T^e}{\partial x^2} + G(T^e - T^l) = 0 \quad \text{in} \quad (0,1)\times(0,\infty), \\[2mm] \mathcal{B}\dfrac{\partial T^l}{\partial t} - G(T^e - T^l) = 0 \quad \text{in} \quad (0,1)\times(0,\infty), \\[2mm] \dfrac{\partial T^e}{\partial x}(0,t) = \dfrac{\partial T^e}{\partial x}(1,t) = \dfrac{\partial T^l}{\partial x}(0,t) = \dfrac{\partial T^l}{\partial x}(1,t) = 0, \quad \text{for all} \quad t \geq 0, \\[2mm] T^e(x,0) = f^e(x), \quad T^l(x,0) = f^l(x), \quad \text{for all} \quad x \in (0,1). \end{cases} \tag{1}$$

and the results presented are the same as those detailed in the presentation abstract.

Acknowledgments

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References

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Structural and microthermal effects on the exponential stabilization of Poroelastic systems

Anderson. J. A. Ramos
Federal University of Pará, Brazil
(joint work with **M. L. P. Nascimento, M. M. Freitas, C. A. S. Nonato**)

This work investigates the well-posedness and asymptotic behavior of a poroelastic system with viscoporosity, viscoelasticity, and microtemperature. We demonstrate the unexpected occurrence of exponential stabilization even when $\gamma\tau - \varepsilon^2 = 0$, in contrast to recent results for thermal systems.

The theory for porous solids with an elastic matrix and voids was introduced by Nunziato & Cowin [1] and later extended by Ieșan [2] to include the effects of temperature and microtemperature. In this work, we study a system of coupled equations that combines structural and microthermal effects, given by

$$\begin{cases} \rho u_{tt} - \mu u_{xx} - b\phi_x - \gamma u_{xxt} - \varepsilon_1 \phi_{xt} = 0 \quad \text{in} \quad (0,\ell)\times(0,\infty), \\ \rho k \phi_{tt} - \delta \phi_{xx} + b u_x + \xi\phi + \tau\phi_t + \varepsilon_2 u_{xt} + d w_x = 0 \quad \text{in} \quad (0,\ell)\times(0,\infty), \\ \alpha w_t - \kappa_4 w_{xx} + \kappa_2 w + d\phi_{xt} = 0 \quad \text{in} \quad (0,\ell)\times(0,\infty), \\ u(0,t) = u(\ell,t) = \phi_x(0,t) = \phi_x(\ell,t) = w(0,t) = w(\ell,t) = 0, \quad \text{for all} \quad t \geq 0, \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x),\ \phi(x,0) = \phi_0(x) \quad \text{in} \quad 0 < x < \ell, \\ \phi_t(x,0) = \phi_1(x),\ w(x,0) = w_0(x) \quad \text{in} \quad 0 < x < \ell. \end{cases} \tag{2}$$

It is well known that assuming the condition $\Sigma := \gamma\tau - \varepsilon^2 = 0$, weakens the dissipation of the poroelastic system (isothermal/thermal), leading to a lack of exponential stability and resulting instead in optimal polynomial decay. However, our results contrast with this scenario by showing that the poroelastic system (2), under the effects of microtemperature, still exhibits exponential decay even under condition $\Sigma = 0$.

Acknowledgments

This work was supported by CNPq (Brazil), Grant No. 313081/2021-2.

References

1. J. W. Nunziato and S. C. Cowin, A nonlinear theory of elastic materials with voids, *Archive for Rational Mechanics and Analysis*, **72** (1979), pp. 175–201.
2. D. Ieşan, A theory of thermoelastic materials with voids, *Acta Mechanica*, **60** (1986), pp. 67–89.

Existence and multiplicity of solution to fourth-order equations with critical Sobolev growth

Angelo Guimarães
Universidade Estadual de Goiás, Brasil
(joint work with **Eduardo. H. Gomes Tavares, Edcarlos Domingos da Silva and Jin-Yun Yuan**)

We investigate a fourth-order elliptic problem of Leray–Lions type involving combined nonlinearities and Sobolev-critical growth, subject to Navier and Dirichlet boundary conditions. The analysis is carried out in two distinct regimes for the perturbation term, namely sublinear and superlinear. In the sublinear case, infinitely many weak solutions are obtained by means of topological variational tools such as Krasnosel'skii's genus and Clark's deformation lemma. In the superlinear case, the existence of a nontrivial solution follows from the Mountain Pass Theorem. The results are further applied to the study of Hamiltonian systems, demonstrating the relevance of fourth-order Leray–Lions equations in both theoretical and applied contexts.

Introduction

Fourth order elliptic equations arise in several physical models, notably in the theory of elastic plates, where the vertical displacement is governed by the Kirchhoff–Love equation involving the biharmonic operator [4]. Motivated by this framework, we study the fourth-order Leray–Lions type problem

$$\Delta(f(x,\Delta u)) = \mu g(x)|u|^{s-1}u + |u|^{q-1}u \quad \text{in } \Omega, \tag{P}$$

subject to either Navier

$$u = \Delta u = 0 \quad \text{on } \partial\Omega,$$

or Dirichlet boundary conditions

$$u = \frac{\partial u}{\partial \eta} = 0 \quad \text{on } \partial\Omega.$$

Here, $\Omega \subset \mathbb{R}^N$ is a bounded domain, $\mu > 0$, $g \in C^1(\Omega)$ is positive, and (p,q) belongs to the critical hyperbola

$$\frac{1}{p+1} + \frac{1}{q+1} = \frac{N-2}{N}.$$

For a fixed $r \in (0,p)$, we consider both sublinear $(0 < s < 1/p)$ and superlinear $(1/r \le s < q)$ perturbations.

The nonlinearity $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is assumed to be continuous, odd and non-decreasing in the second variable, satisfying suitable growth, coercivity and asymptotic conditions, as follows:

(f_1) $f(x,t) \le t^{1/p}$ for every $t > 0$;

(f_2) $\displaystyle\lim_{t\to\infty} \frac{f(x,|t|)}{|t|^{1/p}} = 1$;

(f_3) There exist $t_0 \ge 0$ and positive constants c_p, c_r such that

$$f(x,t) \ge \begin{cases} c_r t^{1/r}, & 0 < t \le t_0, \\ c_p t^{1/p}, & t > t_0; \end{cases}$$

(f_4) For every $(x,t) \in \Omega \times \mathbb{R}$ there exists $c_q > \frac{1}{q+1}$ such that

$$F(x,t) := \int_0^t f(x,s)\,ds \ge \begin{cases} c_q f(x,t)t & \text{sublinear case,} \\ \frac{1}{s+1} f(x,t)t & \text{superlinear case.} \end{cases}$$

Typical examples includes $f(x,t) = |t|^{1/p-1}t$, $f(x,t) = \mathcal{F}_\lambda^{-1}(t)$ where $\mathcal{F}_\lambda(t) = \lambda|t|^{r-1}t + |t|^{p-1}t$, $f(x,t) = |t|^{\frac{1}{p}-1}t\left(1 - \dfrac{1}{2+|t|+|x|}\right)$ and perturbations of these type.

Problem (P) encompasses several classes of nonlinear biharmonic equations, including p-biharmonic, weighted, variable exponent and ϕ-biharmonic operators, and is closely related to the qualitative analysis of Hamiltonian systems (see [1, 3] and the references therein). The variational structure follows the classical Leray–Lions approach [2].

Results

In the sublinear regime, topological methods allow us to detect infinitely many critical points of the energy functional, leading to multiple weak solutions:

Theorem 1. *Under assumptions (f_1)–(f_4) and $0 < s < 1/p$, there exists $\mu_0 > 0$ such that, for every $\mu \in (0, \mu_0)$, problem (P) with either Navier or Dirichlet boundary conditions admits infinitely many weak solutions.*

The proof relies on a truncation argument combined with variational methods.

In the superlinear case, additional dimensional assumptions are required.

(f_5)

$$s+1 > \begin{cases} q-p, & \text{if } \frac{2}{N-2} < p < p_1(N) \\ \frac{(p-1)(q+1)}{p}, & \text{if } p \geq p_2(N), \end{cases}$$

where $p_1(N)$ and $p_2(N)$ are constants that only depends on N.

Theorem 2. *Assume (f_1)–(f_5) and $1/r < s < q$. Then problem (P) admits a weak solution under both Navier and Dirichlet boundary conditions. Moreover, the same conclusion holds for $s = 1/r$ provided that μ is suitable small.*

Under stronger assumptions on the primitive of f, some of the dimensional restrictions can be relaxed.

(f_6) There exist $t_0, c_{p,r} > 0$ such that

$$F(x,t) \leq \tfrac{p}{p+1} f(x,t)t \leq \tfrac{p}{p+1}\left[t^{\frac{p+1}{p}} - c_{p,r}t^{\frac{r+1}{p}}\right], \quad t \geq t_0,\ x \in \Omega.$$

Theorem 3. *Suppose (f_1)–(f_4), (f_6) and $N \geq 7$, then the conclusion of Theorem 2 remains valid.*

The proofs of Theorems 2 and 3 are based on the Mountain Pass Theorem.

Connections with Hamiltonian systems

The fourth-order Leray–Lions problem studied in this work admits a natural reformulation as an elliptic Hamiltonian system. Indeed, for nonlinearities depending only on the Laplacian, the change of variables $v = -f(\Delta u)$ transforms weak solutions of the biharmonic equation with Navier boundary conditions into strong solutions of a coupled second-order system of the type

$$\begin{cases} -\Delta u = f^{-1}(v) & \text{in } \Omega, \\ -\Delta v = \mu g(x)|u|^{s-1}u + |u|^{q-1}u & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega. \end{cases} \tag{HS}$$

This correspondence preserves the variational structure and allows our existence and multiplicity results to be extended to elliptic Hamiltonian systems with Sobolev-critical growth.

In particular, our analysis covers perturbations of critical Lane–Emden systems, including cases with sublinear unilateral terms, which have not been previously addressed in this generality. In the sublinear regime, we obtain infinitely many solutions, extending earlier results that were restricted to purely superlinear interactions. In the superlinear case, our approach complements existing works and recovers known existence results under broader assumptions on the nonlinearities.

Moreover, under additional regularity conditions, the variational framework yields qualitative properties of the associated Hamiltonian solutions, such as improved regularity, positivity of least-energy solutions, and radial symmetry in ball domains. These results highlight the relevance of the fourth-order formulation as a tool for the qualitative and quantitative analysis of elliptic Hamiltonian systems.

Acknowledgments

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On the orbital stability of periodic snoidal-type solutions to the ϕ^4-equation

Beatriz Signori Lonardoni
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(joint work with **Fábio Natali**)

The main purpose of this work is to investigate the global well-posedness and orbital stability of odd periodic traveling waves for the ϕ^4-equation in the Sobolev space of periodic functions with zero mean. The existence of global weak solutions is established by combining semigroup theory with refined energy estimates. Furthermore, the orbital stability is characterized through a Morse Index Theorem applied to the linearized operator, under the zero-mean constraint.

Introduction

We consider the well-known ϕ^4-equation

$$\phi_{tt} - \phi_{xx} - \phi + \phi^3 = 0, \tag{3}$$

where $\phi : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}$ is an L-periodic function at the spatial variable. This model arises in particle and nuclear physics and admits kinks, anti-kinks, and periodic solutions. Our focus is on the odd periodic traveling waves of the form $\phi(x,t) = h(x-ct)$, where $c \in \mathbb{R}$ is the wave speed, and h satisfies the ODE

$$-\omega h'' - h + h^3 = 0, \quad \omega = 1 - c^2.$$

A family of odd periodic solutions is explicitly expressed in terms of the Jacobi elliptic function of snoidal type as

$$h(x) = \frac{\sqrt{2k}}{\sqrt{k^2+1}} sn\left(\frac{4K(k)}{L}\, x; k\right), \quad k \in (0,1), \tag{4}$$

with ω satisfying the equality $\omega = \dfrac{L^2}{16K^2(k)(1+k^2)}$. Here $K(k)$ is the complete elliptic integral of the first kind. It is important to highlight that h is odd and, therefore, has zero mean.

In this context, one of the most significant contributions of our work is the proof of orbital stability in the space $Y = H^1_{per,m} \times L^2_{per,m}$, which lies between $H^1_{per,odd} \times L^2_{per,odd}$ and the full space $H^1_{per} \times L^2_{per}$, where the waves are known to be stable and unstable, respectively (see [2,4]).

Local and global well-posedness

Let us consider the classical Cauchy problem associated with the evolution equation (3)

$$\begin{cases} \phi_{tt} - \phi_{xx} - \phi + \phi^3 = 0, & \text{in } [0,L] \times (0,+\infty), \\ \phi(x,0) = \phi_0(x), \quad \phi_t(x,0) = \phi_1(x), & \text{in } [0,L]. \end{cases} \tag{5}$$

We cannot garantee that the Cauchy problem (5) is locally well-posed in the Sobolev space $H^2_{per,m} \times H^1_{per,m}$ when employing the standard semigroup approach as in [5].

To solve this challenge, let us define $\psi = \partial_x \phi_t$. It is necessary to examine the modified Cauchy problem related to the problem in (5) expressed as

$$\begin{cases} \begin{pmatrix} \phi \\ \psi \end{pmatrix}_t = \begin{pmatrix} \partial_x^{-1} & 0 \\ 0 & \partial_x \end{pmatrix} \begin{pmatrix} 0 & 1 \\ \partial_x^2 & 0 \end{pmatrix} \begin{pmatrix} \phi \\ \psi \end{pmatrix} + \begin{pmatrix} 0 \\ \partial_x(\phi - \phi^3) \end{pmatrix}, & \text{in } [0,L] \times (0,+\infty), \\ \begin{pmatrix} \phi(0) \\ \psi(0) \end{pmatrix} = \begin{pmatrix} \phi_0 \\ \psi_0 \end{pmatrix}, & \text{in } [0,L]. \end{cases} \tag{6}$$

We first need to obtain a local strong solution to the auxiliary problem in (6) by applying the abstract semigroup theory. To this end, we prove initially that the linear (unbounded) operator $A = \begin{pmatrix} \partial_x^{-1} & 0 \\ 0 & \partial_x \end{pmatrix} \begin{pmatrix} 0 & 1 \\ \partial_x^2 & 0 \end{pmatrix}$, defined on $X = H^2_{per,m} \times L^2_{per,m}$ with domain $D(A) = H^3_{per,m} \times H^1_{per,m}$, is a generator of a contraction semigroup on X.

Next, based on Banach's Fixed Point Theorem, we establish the existence of a local strong solution of (6) for initial data $(\phi_0, \psi_0) \in H^3_{per,m} \times H^1_{per,m}$. Moreover, for smoother data $(\phi_0, \phi_1) \in H^3_{per,m} \times H^2_{per,m}$, we prove that ϕ is a smooth solution of the projected equation

$$\phi_{tt} - \phi_{xx} - \phi + \phi^3 - \frac{1}{L}\int_0^L \phi^3 dx = 0.$$

Additionally, using the conserved quantity (energy) given by

$$\mathcal{E}(\phi, \phi_t) = \frac{1}{2}\int_0^L \left[\phi_x^2 + \phi_t^2 - \phi^2 + \frac{\phi^4}{2}\right] dx,$$

we show that the local solution (ϕ, ϕ_t) for the projected problem is global in time in Y, since

$$\int_0^L [\phi_x(x,t)^2 + \phi_t(x,t)^2] dx \leq 2\mathcal{E}(\phi_0, \phi_1) + \frac{L}{2}, \quad L \in (0, 2\pi).$$

Finally, using the classical Poincaré-Wirtinger inequality, we prove the existence and uniqueness of a global weak solution to the Cauchy problem (5), as stated in the following result:

Proposition 4 (Existence of a weak solution). *Let $L \in (0, 2\pi)$ be fixed and consider initial data $(\phi_0, \phi_1) \in Y$. Then there exists a unique global (weak) solution ϕ to the Cauchy problem (5) satisfying $(\phi, \phi_t) \in C([0, +\infty), Y)$.*

Spectral analysis and orbital stability for the ϕ^4−equation

Let $L \in (0, 2\pi)$ be fixed and consider $c \in (-1, 1)$. We first introduce the operators defined by

$$\mathcal{L} = \begin{pmatrix} -\partial_x^2 - 1 + 3h^2 & c\partial_x \\ -c\partial_x & 1 \end{pmatrix}, \qquad \mathcal{L}_\Pi = \mathcal{L} - \begin{pmatrix} \frac{3}{L}\int_0^L h^2 \cdot \, dx & 0 \\ 0 & 0 \end{pmatrix}.$$

As shown in [2, Proposition 3.8], the operator $\mathcal{L}$ has exactly one simple negative eigenvalue, and zero is a simple eigenvalue with eigenfunction (h', ch''). Additionally, the remaining spectrum consists of a discrete set of eigenvalues.

In order to count the number of negative eigenvalues of $\mathcal{L}_\Pi$, we consider the constrained space

$$S = [(1,0), (0,1)] \subset \mathrm{Ker}(\mathcal{L})^\perp = [(h', ch'')]^\perp,$$

which satisfies $\mathcal{L}_\Pi\big|_{S^\perp} = \mathcal{L}$. Next, we define the matrix

$$D = \begin{bmatrix} (\mathcal{L}^{-1}(1,0), (1,0))_{\mathbb{L}^2_{per}} & (\mathcal{L}^{-1}(1,0), (0,1))_{\mathbb{L}^2_{per}} \\ (\mathcal{L}^{-1}(1,0), (0,1))_{\mathbb{L}^2_{per}} & (\mathcal{L}^{-1}(0,1), (0,1))_{\mathbb{L}^2_{per}} \end{bmatrix} = \begin{bmatrix} D_1 & 0 \\ 0 & L \end{bmatrix},$$

where $\mathcal{L}_1 = -\omega\partial_x^2 - 1 + 3h^2$ and $D_1 = (\mathcal{L}_1^{-1}1, 1)_{L^2_{per}}$.

We can construct a function $\tilde{f} \in H^2_{per}$, defined as

$$\tilde{f} = \frac{1}{2\lambda_0\lambda_4\sqrt{1-k^2+k^4}}[(1+k^2+\sqrt{1-k^2+k^4})\lambda_4 f_0 - (1+k^2-\sqrt{1-k^2+k^4})\lambda_0 f_4)],$$

that satisfies $\mathcal{L}_1 \tilde{f} = 1$. Then, for any fixed $L \in (0, 2\pi)$, we have that $D_1 = (\tilde{f}, 1)_{L^2_{per}} < 0$.

According to the **Morse Index Theorem** and since $\mathrm{Ker}(\mathcal{L})$ is simple, we obtain

$$\mathrm{n}(\mathcal{L}_\Pi) = \mathrm{n}(\mathcal{L}) - \mathrm{n}_0 - \mathrm{z}_0 \quad \text{and} \quad \mathrm{z}(\mathcal{L}_\Pi) = \mathrm{z}(\mathcal{L}) + \mathrm{z}_0,$$

where $\mathrm{n}(\mathcal{L})$ and $\mathrm{z}(\mathcal{L})$ denote, respectively, the number of negative eigenvalues of $\mathcal{L}$ and the dimension of its kernel (counting multiplicities), and similarly for $\mathcal{L}_\Pi$. In addition, the numbers n_0 and z_0 are defined as

$$\mathrm{n}_0 = \begin{cases} 1, \text{ if } D_1 < 0, \\ 0, \text{ if } D_1 \geq 0, \end{cases} \qquad \text{and} \qquad \mathrm{z}_0 = \begin{cases} 1, \text{ if } D_1 = 0, \\ 0, \text{ if } D_1 \neq 0. \end{cases}$$

Under these conditions, the following result holds:

Proposition 5. *Let $L \in (0, 2\pi)$ be fixed. The linear operator $\mathcal{L}_\Pi$ has no negative eigenvalues and (h', ch'') is a simple eigenfunction associated with the zero eigenvalue.*

By Proposition 5, we see that $\mathcal{L}_\Pi \geq 0$. This fact ensures the existence of a constant $C > 0$ satisfying

$$(\mathcal{L}(p,q), (p,q))_{\mathbb{L}^2_{per}} = (\mathcal{L}_\Pi(p,q), (p,q))_{\mathbb{L}^2_{per}} \geq C||(p,q)||^2_{\mathbb{L}^2_{per}}, \tag{7}$$

for all $(p,q) \in H^2_{per,m} \times H^1_{per,m}$ such that $((p,q), (h', ch''))_{\mathbb{L}^2_{per}} = 0$, where $\mathbb{L}^2_{per} = L^2_{per} \times L^2_{per}$.

As established in [3] (see also [1]), the coercivity condition in (7) is sufficient to prove that the periodic wave (h, ch') is orbitally stable in Y.

The following statement summarizes our stability result:

Theorem 6 (Orbital stability for the ϕ^4-equation)**.** *Let $L \in (0, 2\pi)$ be fixed. Suppose $c \in (-1, 1)$ and let h be the periodic solution given by (4). Then, the periodic wave (h, ch') is orbitally stable in Y.*

Concluding remarks

In this study, we have developed an alternative framework for analyzing the orbital stability of periodic snoidal waves associated with the ϕ^4-equation. While the instability of such waves in the full energy space is well established, our analysis is carried out within the zero-mean subspace $H^1_{per,m} \times L^2_{per,m}$, where stability arises from the elimination of negative spectral directions. Therefore, our results provide new insights into the behavior of zero-mean periodic waves in the setting of Klein–Gordon-type equations.

Acknowledgments

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Characterization of the uniform exponential stability for a thermo-viscoelastic Timoshenko system with memory

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(joint work with **Eduardo H. G. Tavares, Márcio A. Jorge Silva**)

In this work it is established the characterization of the uniform exponential stability for the thermo-viscoelastic Timoshenko beam system under the Fourier law for heat conduction and memory in a history setting given by

$$\begin{cases} \rho_1\phi_{tt} - \kappa(\phi_x + \psi)_x + \sigma\theta_x = 0 & \text{in } (0,L)\times\mathbb{R}^+, \\ \rho_2\psi_{tt} - b\psi_{xx} + \kappa(\phi_x+\psi) + \displaystyle\int_0^\infty g(s)\psi_{xx}(s)\,ds - \sigma\theta = 0 & \text{in } (0,L)\times\mathbb{R}^+, \\ \rho_3\theta_t - \beta\theta_{xx} + \sigma(\phi_x+\psi)_t = 0 & \text{in } (0,L)\times\mathbb{R}^+, \end{cases}$$

subject to suitable initial-boundary conditions. Here, the unknown functions $\phi = \phi(x,t), \psi = \psi(x,t)$, and $\theta = \theta(x,t)$ represent, respectively, the vertical displacement, the rotation angle, and the temperature deviation of a beam with length $L > 0$.

Motivated by [1, 2], we explore the intrinsic non differentiable assumption on the memory kernel that provides a necessary and sufficient condition $g(\tau+s) \le e^{-\delta\tau}g(s),\ s > 0,\ \tau \ge 0$ for the exponential stability of the whole system. It gives a substantial generalization of the stability results obtained in [3, 4, 5] and surely ties up loose ends about the hypothesis equivalent to exponential stability of the problem.

Acknowledgments

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Extension of the shadowing property to infinite dimensional dynamical systems

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(joint work with **J. M. Arrieta & A. N. Carvalho**)

In this talk, we will present results concerning the Shadowing property. Our main goal is to extend known results about Shadowing from finite-dimensional compact manifolds to infinite-dimensional spaces. As far as we know, the Lipschitz Shadowing property in nonlinear systems typically requires some form of finite dimensionality, even when the phase space is infinite dimensional. For instance, if $\mathcal{T}$ is a Morse–Smale dynamical system defined on a Hilbert space X that possesses an inertial manifold $\mathcal{M}$ (which is finite dimensional), then $\mathcal{T}$ exhibits the Shadowing property in a neighborhood of the attractor $\mathcal{A}$. Motivated by this, we show that it is still possible to obtain the Shadowing property in an infinite-dimensional setting, without relying on the existence of an inertial manifold or reducing the problem to a finite-dimensional case.

Acknowledgments

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Principles of basic research

Eduardo H. Gomes Tavares
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Abstract

In this talk, we will discuss about the critical role of mentorship in developing research autonomy in pure mathematics. We argue that effective supervision is a deliberate pedagogical process of scaffolding and fading. A competent mentor structures the novice's entry into the research frontier, models the heuristics of discovery, and systematically fosters independent problem-solving. The ultimate objective of this mentorship is to transform passive learners into active, self-reliant investigators. Thus, successful guidance is measured by its own obsolescence, culminating in the emergence of an independent mathematical researcher.

Existence of an inertial manifold for a nonlocal quasilinear scalar one-dimensional problem

Estefani Moraes Moreira
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(joint work with **Xiaoqing Yang, Alexandre N. Carvalho**)

Abstract

First, we consider the existence of a global attractor for a nonlocal quasilinear problem set in L^2. Then, we show that the global attractor can be given as a graph of a Lipschitz function defined on a finite dimensional space.

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On the stability of Bresse systems: observability inequality via resolvent equation

Gabriel E. Bittencourt Moraes
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(joint work with **Marcio A. Jorge da Silva**)

We discuss the asymptotic stability (exponential and polynomial) of Bresse-type systems coupled with Fourier thermal dissipations. The centerpiece of this talk is the application of an observability inequality developed in [1], which allows bypassing certain limitations imposed by the boundary conditions of the Bresse system through local estimates on subintervals of the domain $[0, L]$. We demonstrate how this technique enables the extension of energy estimates to the entire domain by employing cut-off functions. Furthermore, we present recent works $[1, 2, 3]$ where this result proved essential in obtaining polynomial and exponential stability results for Bresse-type systems with thermoelastic coupling in two components (shear force, bending moment, and axial force). Finally, we discuss perspectives for extending these results to models with a single dissipation, as well as other dissipative mechanisms in Bresse systems, such as frictional damping, delay, and memory effects.

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Non-autonomous Dynamical Systems with Applications to Non-cylindrical Problems in PDEs

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Abstract

In this talk, I will introduce the notion of dynamical systems evolving on time-varying phase spaces, emphasizing fundamental results concerning pullback dynamics. We analyze the relationship between this setting and partial differential equations on non-cylindrical domains. Classical examples will be addressed, including applications to parabolic problems such as the Navier–Stokes equations and nonlinear reaction–diffusion models.

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Aplicação de Equações Diferenciais Parciais no Realce de Imagens de Impressões Digitais

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(joint work with **Neyva Maria Lopes Romeiro**)

Este artigo analisa a aplicação de Equações Diferenciais Parciais (EDPs) no realce, na redução de ruído e na segmentação de imagens de impressões digitais, com ênfase na equação do calor isotrópica e em modelos anisotrópicos do tipo Perona–Malik, conforme proposto no trabalho clássico de Perona e Malik. O objetivo é aprimorar a visualização das cristas e dos vales papilares, favorecendo a extração de minúcias em aplicações forenses, em consonância com estudos consolidados na área de reconhecimento de impressões digitais. Foram avaliadas soluções numéricas obtidas por discretizações em diferenças finitas, bem como combinações de métodos clássicos de processamento digital de imagens, incluindo CLAHE, transformadas wavelet, filtros de nitidez e técnicas de redução de ruído, amplamente descritas na literatura de Gonzalez. Os resultados indicam que as abordagens baseadas em EDPs preservam bordas relevantes, reduzem ruídos e promovem aumento da nitidez estrutural, contribuindo para análises papiloscópicas mais robustas, confiáveis e estáveis.

Introdução

A papiloscopia depende da qualidade visual das cristas papilares para a identificação humana, porém imagens periciais frequentemente apresentam ruídos, baixa nitidez e contraste irregular, dificultando a análise das minúcias.

Nesse contexto, métodos baseados em Equações Diferenciais Parciais (EDPs) destacam-se por permitir suavização controlada e preservação de bordas, como no modelo anisotrópico de Perona–Malik, em contraste com abordagens isotrópicas clássicas, como a equação do calor.

Este trabalho avalia a aplicação dessas técnicas na melhoria de imagens de impressões digitais, utilizando as métricas PSNR (Peak Signal-to-Noise Ratio) e SSIM (Structural Similarity Index) para análise do realce das cristas papilares em contextos periciais.

Fundamentos Matemáticos

Equação do Calor

A equação do calor é um modelo clássico de difusão isotrópica, amplamente utilizado no processamento de imagens para suavização e redução de ruído. Em duas dimensões espaciais, sua forma contínua é dada por:

$$\frac{\partial u(x,y,t)}{\partial t} = \alpha\, \Delta u(x,y,t),$$

onde o operador Laplaciano bidimensional é definido por:

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}.$$

A discretização explícita por diferenças finitas resulta na seguinte atualização iterativa:

$$u_{i,j}^{n+1} = u_{i,j}^n + \lambda \left(u_{i+1,j}^n + u_{i-1,j}^n + u_{i,j+1}^n + u_{i,j-1}^n - 4u_{i,j}^n \right),$$

Para garantir a estabilidade do esquema explícito, deve-se respeitar a condição:

$$\lambda \leq \frac{1}{4}.$$

Embora eficaz na redução de ruído, a equação do calor promove difusão uniforme em todas as direções, o que pode resultar em suavização excessiva e perda de bordas relevantes, especialmente em aplicações forenses que exigem preservação de detalhes estruturais.

Modelo de Difusão Anisotrópica de Perona–Malik

Com o objetivo de superar as limitações da difusão isotrópica, Perona e Malik propuseram um modelo de difusão anisotrópica que permite suavização seletiva da imagem, preservando descontinuidades associadas a bordas. O modelo é descrito pela seguinte equação diferencial parcial:

$$\frac{\partial u}{\partial t} = \nabla \cdot (g(|\nabla u|)\nabla u)\,,$$

Uma escolha comum para a função de difusão é:

$$g(s) = \frac{1}{1 + \left(\frac{s}{k}\right)^2},$$

Esse comportamento confirma a adequação do modelo de Perona–Malik ao processamento de impressões digitais, ao promover suavização seletiva com preservação das cristas papilares e das minúcias. Os resultados, corroborados por comparações visuais e histogramas, indicam melhor preservação do contraste e redução de ruído em relação à difusão isotrópica e aos métodos clássicos.

Resultados e Avaliação

Neste trabalho, as imagens processadas foram avaliadas por análises quantitativas e qualitativas, incluindo inspeção visual e histogramas. Os resultados demonstram que a difusão anisotrópica supera a difusão isotrópica e os métodos clássicos, ao preservar as cristas papilares, aumentar o contraste entre cristas e vales e promover distribuição de intensidades mais equilibrada, com redução de ruído e preservação das bordas relevantes à identificação papiloscópica.

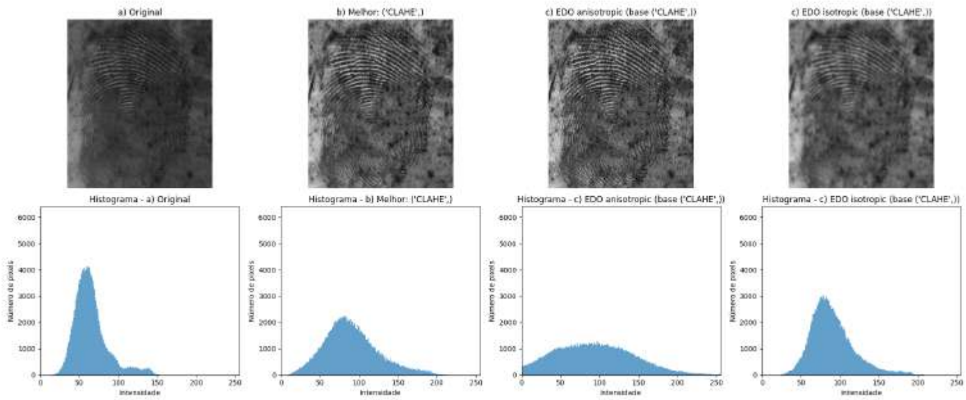

Figura 2: Comparação entre (a) imagem original, (b) imagem processada por filtro tradicional (CLAHE), (c) difusão anisotrópica (Perona–Malik) e difusão isotrópica (equação do calor).

A Figura 2 compara os métodos de processamento de impressões digitais. O CLAHE amplia o contraste global, porém mantém ruído local, enquanto a difusão anisotrópica de Perona–Malik preserva cristas e bordas. Em contraste, a difusão isotrópica causa suavização excessiva, evidenciando a superioridade dos modelos anisotrópicos para o realce forense.

Conclusão

A aplicação de Equações Diferenciais Parciais mostrou-se eficaz no realce forense de impressões digitais, ao melhorar a visualização das cristas papilares e a confiabilidade das análises. Contudo, a metodologia ainda requer aprimoramento na calibração e otimização dos parâmetros anisotrópicos, sendo necessárias etapas adicionais de validação e análise quantitativa para consolidar sua aplicação prática em contextos forenses.

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Comparison of long-time dynamics between the Bresse System and the Timoshenko–wave system

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(joint work with **Rodrigo Nunes Monteiro**)

The Bresse system models the dynamics of beams with small initial curvature $\ell > 0$ and consists of three coupled wave-type equations. When $\ell = 0$, it reduces to the Timoshenko system and a wave equation. Motivated by the singular limit results connecting these models, we propose a different approach that avoids projection arguments proposed by [4].

We consider a Bresse system with a nonlinear term acting on the shear angle and frictional damping acting on both the shear angle and the longitudinal displacement. First, we establish the well-posedness of the system for each $\ell > 0$. Then, as $\ell \to 0$, we show that the solutions converge to weak solutions of the Timoshenko system and the wave equation, allowing a direct comparison of their long-time dynamics.

Using the quasi-stability method, we prove the existence of a compact global attractor with finite fractal dimension for each $\ell > 0$. Finally, we demonstrate the upper semicontinuity of the family of attractors as $\ell \to 0$, where the limit is given by the Cartesian product of the attractor of the Timoshenko system (see e.g. [3]) and the trivial equilibrium of the wave equation.

Acknowledgments

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Timoshenko-Boltzmann systems with non-smooth memory kernels

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(joint work with **Eduardo H. Gomes Tavares, Marcio A. Jorge Silva, Jin-Yun Yuan**)

The exponential characterization of a partially dissipative Timoshenko system with memory effects acting on the shear component is addressed in this work. A wider class of admissible kernels that may exhibit flat zones and jump discontinuities is explored. Then, by the introduction of a novel condition, referred to as the shape condition, it is proven that the previous classical criteria for the system's exponential characterization are no longer sufficient in our extended setting. Here, the geometric structure of the kernel, particularly its flatness and the distribution of discontinuities, is seen to play a crucial role in determining the characterization of exponential stability. Hence, a refined characterization is established in terms of three simultaneous conditions: dissipation, equal wave speeds, and the shape condition. The results not only generalize previous findings based on smooth kernels but also offer a deeper understanding of the interplay between the geometric aspects of the memory kernel and the system's asymptotic behavior.

Description of the problem and Well-Posedness

Taking into account the viscoelastic Timoshenko-Boltzmann problem with past history where the memory effects act solely on the shear force, cf. [4, Appendix A], we consider the following system of integro-differential equations for thin beams of length $L = \pi$:

$$\begin{cases} \rho_1 \phi_{tt} - \kappa(\phi_x + \psi)_x + \kappa \displaystyle\int_{-\infty}^{t} \mu(t-s)(\phi_x + \psi)_x(s)\, ds = 0 \text{ in } (0,\pi) \times \mathbb{R}^+, \\ \rho_2 \psi_{tt} - b\psi_{xx} + \kappa(\phi_x + \psi) - \kappa \displaystyle\int_{-\infty}^{t} \mu(t-s)(\phi_x + \psi)(s)\, ds = 0 \text{ in } (0,\pi) \times \mathbb{R}^+, \end{cases} \tag{8}$$

where ρ_1, ρ_2, b, κ are positive constants with structural physical meanings, μ stands for the memory kernel, and $\phi := \phi(x,t)$ and $\psi := \psi(x,t)$ represent the vertical displacement and rotation angle at $x \in [0,\pi]$ and time $t \geq 0$, respectively. Here, we supplement (8) with the boundary conditions

$$\phi(0,t) = \phi(\pi,t) = \psi_x(0,t) = \psi_x(\pi,t) = 0, \quad t \in \mathbb{R}, \tag{9}$$

and initial (also compatibility) data

$$\begin{cases} \phi(x,0) = \phi_0(x),\ \phi_t(x,0) = \phi_1(x), \quad x \in (0,\pi), \\ \psi(x,0) = \psi_0(x),\ \psi_t(x,0) = \psi_1(x), \quad x \in (0,\pi), \\ \phi(x,t) + \widetilde{\psi}(x,t) = p_0(x,t), \quad (x,t) \in (0,\pi) \times (-\infty, 0), \end{cases} \tag{10}$$

with notation

$$\widetilde{h}(x) := \int_0^x h(y)\, dy.$$

After a suitable introduction of the so-called *displacement history variable*, problem (8)-(10) can be converted into an equivalent autonomous abstract Cauchy problem over a properly Hilbert phase space $\mathcal{H}$, which is well-posed and its solutions generate a C_0-semigroup of contractions $S(t)$ in $\mathcal{H}$.

DES-Exponential Characterization

In recent work [4, Theorem 3.3], assuming that $\mu : \mathbb{R}^+ \to \mathbb{R}^+$ is a nonincreasing, absolutely continuous, and summable function with

$$l := \int_0^\infty \mu(s)\, ds \in (0,1) \quad \text{and} \quad \mu(0) := \lim_{s\to 0^+} \mu(s) < \infty,$$

the authors have proved that the C_0-semigroup of contractions $S(t)$ is exponentially stable if and only if the two conditions hold:

(D) Dissipative condition: there exist $\delta > 0$ and $C \geq 1$ such that

$$\mu(t+s) \leq Ce^{-\delta t}\mu(s) \tag{11}$$

for every $t > 0$ and almost every $s > 0$;

(E) Equality of the wave speeds: the coefficients of (8) satisfy

$$\chi_0 := \frac{\kappa}{\rho_1} - \frac{b}{\rho_2} = 0. \tag{12}$$

We call this result a **DE**-exponential characterization for (8)–(10). From this perspective, motivated by the works [1, 2, 5], we bring a complementary viewpoint for the stability characterization of (8)–(10). Indeed, we consider a **non-absolutely continuous** memory kernel μ called *admissible kernel*, i.e., μ is a non-increasing, summable function $\mu : \mathbb{R}^+ \to [0,\infty)$, with $l \in (0,1)$, satisfying the property: there exists a strictly increasing sequence (s_n), with $s_0 = 0$, either finite (possibly reduced to s_0 only) or converging to $s_\infty \in (0,\infty]$, such that μ has jumps at $s = s_n$ and is absolutely continuous on each interval (s_{n-1}, s_n) for every $n \in \mathbb{N}$, as well as on (s_∞, ∞) when defined.

Now, conditions **(D)** and **(E)** will not be sufficient for the exponential characterization of (8)–(10) and this fact requires one more (and new) condition in the full characterization of stability for (8)–(10). Observing that the complete geometric property of such a kernel has not yet been investigated in depth, we consider the notion of the *shape condition* for the memory kernel μ. More precisely, by defining the *flatness set* of μ as

$$\mathcal{F}_\mu := \left\{ s \in \mathbb{R}^+ \mid \mu'(s) = 0,\ \mu(s) > 0 \right\},$$

and its *flatness rate* as

$$\mathfrak{F}_\mu := \frac{1}{l} \int_{\mathcal{F}_\mu} \mu(s)\, ds \in [0,1], \tag{13}$$

we set the non-flatness condition:

(S) *Shape condition of μ: $\mathfrak{F}_\mu < 1$.*

Unlike **DE**-exponential characterization given above, we characterize the exponential stability of (8)–(10) by means of **(D)**, **(E)**, and **(S)**. Otherwise, when the shape condition fails, namely, $\mathfrak{F}_\mu = 1$ and proper jump-conditions for μ is regarded, we shall show lack of exponential stability for (8)–(10) even so conditions **(D)** and **(E)** hold simultaneously.

The main result of this work reads as follows.

Theorem 7 (**DES**-Exponential Characterization)**.** *Let μ be an admissible kernel satisfying $\mu(0) := \lim_{s\to 0^+} \mu(s) < +\infty$, and one of the conditions below:*

(i) $\mathcal{R} \cap \mathbb{Q} = \{0\}$;

(ii) $\dfrac{s_j}{4\pi}\sqrt{\dfrac{\kappa}{\rho_2}} \in \mathbb{N}$, *for every* $j \in \mathbb{N}$.

Then, $S(t)$ *is exponentially stable if and only if the following conditions hold simultaneously:*

(D) *Dissipative condition:* μ *satisfies* (11) *for some* $\delta > 0$*;*

(E) *Equal Wave speeds of propagation:* (12) *holds;*

(S) *Shape of* μ*:* $\mathfrak{F}_\mu < 1$.

Here, we are denoting by $\mathcal{R}$ the set of all finite linear combinations, including the trivial ones, over the rational field of the elements

$$\frac{s_j}{\pi}\sqrt{\frac{b}{\rho_2}}.$$

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Mathematical modeling for the analysis of PDEs

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Abstract

We will discuss how skills in mathematical modeling can contribute to new and relevant developments in the analysis of PDEs.

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Control Systems on Lie Groups

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(joint work with **Túlio Oliveira de Carvalho**)

This study explores the controllability properties of systems described by evolution equations on Lie groups, particularly right-invariant systems of the form $\frac{dx}{dt} = X_0(x) + \sum u_i(t)X_i(x)$. The classical control theory is typically restricted to vector spaces, this work extends to group manifolds motivated by physical applications such as the orientation of rigid bodies. The main focus is to determine necessary and sufficient conditions for the system to be controllable, ie., the set of points attainable from the identity covers the entire group.

Introduction

Unlike classical control theory, which assumes the state space is a vector space, this work considers systems where the state evolves on a differentiable manifold with a group structure, we exploit the algebraic structure of the space to obtain global results on controllability.

The control system is described by the differential equation:

$$\frac{dx}{dt}(t) = X_0(x(t)) + \sum_{i=1}^{m} u_i(t)X_i(x(t)) \quad (*)$$

where $X_0, \ldots, X_m$ are right-invariant vector fields on G and $u = (u_1, \ldots, u_m)$ belongs to the class of admissible controls U.

Definitions

Let G be a Lie group and $L(G)$ its Lie algebra, which is the set of right-invariant vector fields. Admissible controls U are classified into three categories:

- **Unrestricted (U_u):** Measurable and locally bounded functions defined on the interval $[0, \infty)$ with values in $\mathbb{R}^m$.
- **Restricted (U_r):** Functions with values limited to the cube $\{x \in \mathbf{R}^m : |x_i| \leqslant 1, i = 1, \ldots, m\}$
- **Bang-Bang (U_b):** Piecewise constant functions defined on $[0, \infty)$ with values in $\mathbf{R}^m$ such that the components of its elements only take values 1 and -1.

The system described by $(*)$ - we term right-invariant - possesses unique solutions defined for $t \in [0, \infty)$ for any initial condition $g \in G$, such that $\frac{dx}{dt}(0) = g$. We define $A(g, t)$ as the set of points attainable from g at time t, and $\mathbf{A}(g)$ as the union of all attainable sets for $t \in [0, \infty)$.

A necessary condition for controllability is that the set $\mathbf{A}(e)$ of points reachable from the identity of G be a subgroup of G. Thus, the controllability problem reduces to two fundamental questions:

(a) When is $\mathbf{A}(e)$ a subgroup?, and

(b) If $\mathbf{A}(e)$ is a subgroup, when is $\mathbf{A}(e) = G$?

Associated Subalgebras

To every right-invariant control system, we shall associate the following three Lie subalgebras of $L(G)$:

- $\mathbf{L}$: The subalgebra generated by all vector fields $\{X_0, \ldots, X_m\}$.
- $\mathbf{L}_0$: The ideal of $\mathbf{L}$ generated by $\{X_1, \ldots, X_m\}$.
- L: The subalgebra generated only by the control fields $\{X_1, \ldots, X_m\}$.

Let $\mathbf{S}$, $\mathbf{S}_0$, and S be the connected Lie subgroups corresponding to these algebras. It is trivial that $L \subset \mathbf{L}_0 \subset \mathbf{L}$ and $S \subset \mathbf{S}_0 \subset \mathbf{S}$. Furthermore, $\mathbf{L}_0$ is a subspace of $\mathbf{L}$ with codimension less than or equal to 1 and $\mathbf{S}_0$ is a normal subgroup of $\mathbf{S}$.

A relevant property is that the attainable set $\mathbf{A}(e)$ is always contained in $\mathbf{S}$. More importantly, if $\mathbf{A}(e)$ is a subgroup of G, then $\mathbf{A}(e) = \mathbf{S}$. Therefore, the system will be controllable if and only if $\mathbf{A}(e)$ is a subgroup, G is connected and $\mathbf{L}$ is the Lie algebra of G.

Specific Cases

The Homogeneous Case

A system is named homogeneous if $X_0 = 0$. In this case, we have the following result:

Theorem 8. *Let $(\mathbf{X}, U)$ be a homogeneous right-invariant control system on G. Then the set attainable from the identity is the subgroup $\mathbf{S}$. Moreover, if U is unrestricted then, for each $T > 0$, $A(e, T) = \mathbf{A}(e) = \mathbf{S}$.*

This implies that, for homogeneous systems:

- $\mathbf{A}(e)$ is always a subgroup of G.
- If controls are unrestricted, any point in $\mathbf{S}$ can be reached from the identity in an arbitrarily short time.
- The attainable set is the same for unrestricted, restricted, or "bang-bang" controls, possibly at a later time for the bang-bang case.

The Compact Case

For the compact case, we have the next important result:

Theorem 9. *Let $(\mathbf{X}, U)$ be a right-invariant control system on G. Assume that the subgroup $\mathbf{S}$ is compact. Then*

(i) $\mathbf{A}(e) = \mathbf{S}$.

(ii) There exists $T > 0$ such that $\mathbf{A}(e, T) = \mathbf{A}(e)$.

This means compactness guarantees controllability for the subgroup. Additionally, in the compact case, there exists a time $T > 0$ such that all points in $\mathbf{A}(e)$ can be reached in a time less than or equal to T.

The Non-Compact Case

If $\mathbf{S}$ is not compact or the system is not homogeneous, controllability is not guaranteed uniquely by the Lie algebra. The following theorem shows that certain recurrence properties will ensure controllability:

Theorem 10. *Let $(\mathbf{X}, U)$ be a right-invariant control system on G with $U = U_u$. If there exists a constant control u and a sequence of positive numbers $\{t_n\}$ with $t_n \geqslant \epsilon > 0$ for some ϵ, with the property that $\lim \pi(e, u, t_n)$ exists and belongs to $\bar{S}$ (the closure is relative to $\mathbf{S}$), then $\mathbf{A}(e) = \mathbf{S}$.*

The subesequent corollary is immediate:

Corollary 11. *Let $(\mathbf{X}, U)$ be a right-invariant control system on G with $U = U_u$. If there exists a constant control u such that $t \to \pi(e, u, t)$ is periodic, then $\mathbf{A}(e) = \mathbf{S}$.*

Controllability Criteria

We now state the two main controllability criteria:

Theorem 12. *A necessary condition for (X, U) to be controllable is that G be connected and that $\mathbf{L} = L(G)$. If G is compact, or if the system is homogeneous, this condition is also sufficient.*

Finally, for systems where $L = \mathbf{L}_0$, the following result holds:

Theorem 13. *Assume that the necessary conditions of Theorem 5 hold, and that* (i) *$U = U_u$, and* (ii) *$L = \mathbf{L}_0$ (or, equivalently, L is an ideal of $\mathbf{L}$). Then $(\mathbf{X}, U)$ is controllable if and only if $\exp(TX_0)$ belongs to S $(= \mathbf{S}_0)$ for some $T \neq 0$.*

Conclusion

This work has established a way for analyzing the controllability of right-invariant systems on Lie groups, extending classical results previously limited to vector spaces. Certainly the structure of the Lie algebra $\mathbf{L}$ determines of controllability in $\mathbf{S}$, but compactness or some recurrence are still necessary to guarantee that system can cover all the subgroup.

Acknowledgments

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Semilinear Evolution Equations with time-dependent linear operators: existence of pullback attractors and permanence under perturbations

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(joint work with **Caraballo, T.; Nascimento, M.J.D; Schiabel, K.**)

Abstract

In this work we consider parabolic equations of the form

$$(u_\varepsilon)_t + A_\varepsilon(t)u_\varepsilon = F_\varepsilon(t, u_\varepsilon),$$

where ε is a parameter in $[0, \varepsilon_0)$ and $\{A_\varepsilon(t),\ t \in \mathbb{R}\}$ is a family of uniformly sectorial operators. As $\varepsilon \to 0^+$, we assume that the equation converges to

$$u_t + A_0(t)u = F_0(t, u).$$

The time-dependence found on the linear operators $A_\varepsilon(t)$ implies that linear process is the central object to obtain solutions via variation of constants formula. Under suitable conditions on the family $A_\varepsilon(t)$ and on its convergence to $A_0(t)$ when $\varepsilon \to 0^+$, we obtain a Trotter-Kato type Approximation Theorem for the linear process $U_\varepsilon(t, \tau)$ associated to $A_\varepsilon(t)$, estimating its convergence to the linear process $U_0(t, \tau)$ associated to $A_0(t)$. Through the variation of constants formula and assuming that F_ε converges to F_0, we analyze how this linear process convergence is transferred to the solution of the semilinear equation. We illustrate the ideas in two examples. First a reaction-diffusion equation in a bounded smooth domain and then a nonautonomous fractional strongly damped wave equation.
Keywords: Nonautonomous parabolic problems, time-dependent linear operators, perturbed problems, convergence of linear process, convergence of solutions.

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Random Quasi-Stability and Long-Time Dynamics of Stochastic Piezoelectric Systems with Magnetic and Thermal Effects

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(joint work with **Renhai Wang, Marcio A. Jorge Silva, Santiago A. S. Benites**)

We investigate the *random* quasi-stability and long-time dynamics for a class of nonlinear stochastic piezoelectric systems incorporating magnetic effects and thermal dissipation via Fourier's law. We establish the global well-posedness of the pathwise systems transformed from the stochastic equations, and prove that the associated random dynamical system possesses a unique pullback random attractor. Despite the presence of nonlinearities with arbitrary polynomial growth and *non-globally Lipschitz* behavior, we show that the random attractor has a finite and uniform fractal dimension by developing an *improved quasi-stability method* combined with Birkhoff's ergodic theorem. This enables us to resolve an open question posed by Chueshov and Schmalfuß [1] concerning a random coefficient with possibly uncontrolled growth behavior in the quasi-stability estimates in the context of stochastic piezoelectric systems. By the uniform quasi-stability estimates with respect to the noise intensity, we develop a new argument to establish the upper semicontinuity of attractors as the noise intensity vanishes, even without higher regularity assumptions. Our methods can be extended to a broad class of hyperbolic-type stochastic PDEs with additive white noise, linear multiplicative noise, and nonlinear colored noise.

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Navier-Stokes Equations on Thin Domains

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In this work, we consider the Navier-Stokes equations in the thin domain

$$\mathbb{R}^{N-1} \times (0, \varepsilon),$$

where $N \geq 2$ and $\varepsilon > 0$ denotes the thickness of the domain. We work in the space $\mathcal{PM}^{1,1/2}$, which is based on the Fourier transform, continuous in $\mathbb{R}^N$ and periodic in the last coordinate. Within this framework, we obtain a global well-posedness result for mild solutions when ε is sufficiently small, depending on the size of the initial data, which may be arbitrarily large in the space $\mathcal{PM}^{1,1/2}$.

One motivation for studying this type of domain is due to the fact that the $2D$ Navier-Stokes equations has a regular global solution for any regular initial data (see, for example, [1]). Thus, for $N = 3$ one may expect that the fluid in a thin domain has little additional space (in relation to the plane) arising from the third dimension to allow for a singularity formation dynamics, and then we can expect better results of global well-posedness.

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Comparison of the numerical accuracy of triangular and quadrangular meshes in the finite volume method

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(joint work with **Gylles Ricardo Ströher**)

Elliptic partial differential equations (EPDEs) are fundamental for representing physical equilibrium problems. Given the complexity of obtaining analytical solutions in irregular domains, the finite volume method (FVM) is an effective tool due to its versatility. This study aims to analyze the numerical accuracy of the FVM in solving a two-dimensional PDE, comparing the performance of meshes composed of triangular and quadrangular elements under specific discretization conditions.

Discretization was performed using gradient reconstruction on the faces through the "diamond" scheme, which applies the Green-Gauss Method to obtain a second-order approximation. As for the boundary volumes, the midpoint of the contour face was used as a pseudo-centroid, allowing the application of the same methodology as for the internal volumes and integrating the boundary conditions in the calculation of ϕ on the face. The resulting linear system was solved by the Gauss-Seidel method.

To enable comparison, simulations were performed in a square geometric computational domain, using meshes with approximate volumes (4096 squares for 4165 triangles), allowing a direct assessment of how the shape of the control volume influences the accuracy of the solution.

The results obtained validate the conclusions of Juretić and Gosman [1], who attribute the superiority of quadrilaterals to their symmetrical geometry: the truncation error generated on one face tends to be canceled by the opposite face of the same magnitude. In triangular meshes, the absence of pairs of parallel faces prevents this cancellation, resulting in larger errors even in more refined meshes. In general, it was confirmed that quadrangular meshes offer better computational efficiency and numerical accuracy for the case studied.

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Attractors for a two-dimensional Lamé system with exponential nonlinearity

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(joint work with **Marcio A. Jorge da Silva**)

This work studies the dynamics and long-time behavior of a two-dimensional Lamé system with nonlinear vector-valued source terms, formulated as a semilinear hyperbolic system

$$\begin{cases} \partial_{tt}\mathbf{u} - \Delta_e \mathbf{u} + \alpha\partial_t \mathbf{u} + \mathbf{f}(\mathbf{u}) = \mathbf{h} \quad \text{in} \quad \Omega \times \mathbb{R}^+, \\ \mathbf{u} = 0 \quad \text{on} \quad \partial\Omega \times \mathbb{R}^+, \mathbf{u}(0) = \mathbf{u}_0, \;\; \partial_t \mathbf{u}(0) = \mathbf{u}_1 \;\; \text{in} \;\; \Omega, \end{cases} \tag{14}$$

where $\Omega \subset \mathbb{R}^2$ is a bounded domain with smooth boundary $\partial\Omega$, $\mathbf{u} = (u_1, u_2)$ is the unknown displacement vector solution depending on (x,t), with $x = (x_1, x_2) \in \Omega$ and $t \geq 0$, $\alpha > 0$ is the damping coefficient, $\mathbf{h} = \mathbf{h}(x)$ is a given external force, $\mathbf{f} : \mathbb{R}^2 \to \mathbb{R}^2$ is a vector-valued nonlinear perturbation, and Δ_e stands for the elastic Lamé operator

$$\Delta_e \mathbf{u} := \mu\Delta\mathbf{u} + (\lambda + \mu)\nabla \mathrm{div}\mathbf{u}, \;\; \mu > 0, \, \lambda + \mu \geq 0. \tag{15}$$

The evolution problem (14) has been first studied in the three dimension ($\Omega \subset \mathbb{R}^3$) and later its generalizations in higher dimensions ($\Omega \subset \mathbb{R}^n$, $n \geq 3$), and our main goal is to address the Lamé system (14) for the first time in the two-dimensional scenario specifically with fully coupled vector-valued nonlinear perturbation $\mathbf{f}(\mathbf{u}) = (f_1(\mathbf{u}), f_2(\mathbf{u}))$ leading to exponential growth conditions such as: *given a constant $\gamma > 0$ there exists a constant $C_\gamma > 0$ such that* $|\mathbf{f}(\mathbf{u})| \leq C_\gamma |\mathbf{u}| \, e^{2\gamma|\mathbf{u}|^2}$.

The contributions of this work relies on the fact that all qualitative properties obtained for the dynamical system $(\mathcal{H}, S(t))$ corresponding to problem (14) are considered under the exponential setting for $\mathbf{f}(\mathbf{u})$, and we prove the important quasi-stability and smoothing properties, which are crucial to conclude the existence of global and time dependent exponential attractors, by giving a full dynamic analysis of (14).

Acknowledgments

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On the stabilization of twisted Bresse systems

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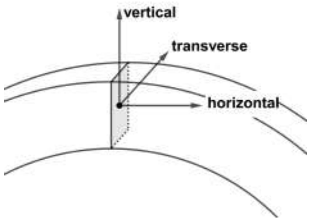
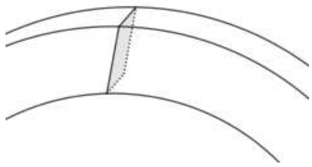
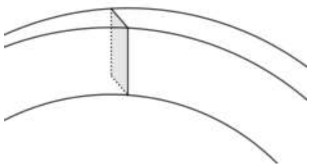
The Bresse system is a well-known PDE model that describes the behavior of a thin curved beam in terms of three variables: the vertical displacement, the horizontal displacement and the angle of rotation about the transverse axis. The Bresse system is a generalization of the Timoshenko system in the sense that the Timoshenko system neglects the horizontal displacement. This talk presents some results on the stabilization of a broader generalization, first studied in [1], which incorporates torsional effects by introducing, according to [2], two additional variables: the transverse displacement and the angle of rotation about the vertical axis (see Figure 3).



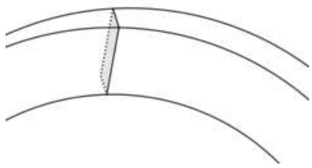
Cross section and directions of displacements.



Rotation about the transverse axis.



Rotation about the vertical axis.



Combination of both rotations.

Figura 3: Directions of displacements and rotations in the twisted Bresse system.

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Asymptotic dynamics of an extensible beam model with fractional nonlocal damping

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(joint work with **Marcio A. Jorge da Silva**)

Let $\Omega \subset \mathbb{R}^d$ be an open set with sufficiently smooth boundary $\Gamma = \partial\Omega$. We consider a beam equation with nonlocal fractional damping given by

$$u_{tt} + \Delta^2 u + M\left(\int_\Omega |(-\Delta)^{\frac{\sigma}{2}} u|^2 \, \mathrm{d}x\right)(-\Delta)^\sigma u + \left(\int_\Omega |(-\Delta)^{\frac{\delta}{2}} u_t|^2 \, \mathrm{d}x\right)^{\frac{\rho}{2}} (-\Delta)^\delta u_t + f(u) = 0, \quad (16)$$

with initial conditions

$$u(x,0) = u_0(x), \ u_t(x,0) = u_1(x), \ x \in \Omega. \quad (17)$$

and Dirichlet boundary conditions

$$u|_{\partial\Omega} = 0, \ \Delta u|_{\partial\Omega} = 0. \quad (18)$$

Throughout this work, the parameters $\sigma, \delta \in [0,1]$ and $\rho \in \mathbb{R}^+$, $f \in C^1(\mathbb{R})$ is the source term and $M \in C^1(\mathbb{R})$ is the extensibility coefficient. We suppose that f and M satisfy the standard assumptions. This work main contributions are well-posedness of problem (16)-(18), existence of global attractor $\mathbf{A}_\rho$ and upper semicontinuity of the family $\{\mathbf{A}_\rho\}_{\rho \in \mathbb{R}_+}$ with relation to ρ.

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Lack of exponential stability for fully dissipative Timoshenko systems

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(joint work with **Leonardo Gustavo Ronchin Alves**)

In general, fully dissipative Timoshenko systems, in the sense that dissipation acts in each equation of the system, are known to be exponentially stable. In this work, we show that this behavior does not always hold. More precisely, we introduce a dissipative mechanism which, although acting on all components of the system, does not guarantee exponential decay of the associated energy. Instead, we prove that the solutions decay at a rational rate. This result highlights the delicate interplay between the structure of the dissipation and the long-time dynamics of Timoshenko systems, and shows that full dissipation alone is not sufficient to ensure exponential stability.

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Attractors for the Wave Equation With Critical Growth Nonlinearities

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(joint work with **Irena Lasiecka**)

Abstract

The wave equation with energy-critical nonlinear sources and dissipation in a bounded three-dimensional domain is considered. It is shown that the associated dynamical system admits a global attractor. Under the additional assumption of strong monotonicity of the damping at the origin, this attractor is also finite-dimensional and smooth. Moreover, the existence of an exponential attractor is established. In order to handle "energy criticality" of both sources and damping, the methods used depend on enhanced dissipation [2], energy *identity* for weak solutions [4], an adaptation of Ball's method [1], and the theory of quasi-stable systems [3].

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Asymptotic Behavior of Nonlinear Quintic Defocusing KGS with a Localized Damping

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We are concerned with the existence and asymptotic behavior of the quintic defocusing Klein-Gordon-Schrödinger (KGS) equation posed in a 3-dimensional bounded domain subject to two types of locally distributed damping. To our the best knowledge, it is the first result handling with the cubic $|\psi|^2\,\psi$ jointly with the quintic term improving the results of linear KGS equation given, for example, in [1, 2, 4, 9, 10] and references therein. Moreover, we prove that the corresponding energy functional goes to zero as time goes to infinity by employing Strichartz estimate given by Blair et al. [3], the unique continuation property and observability inequalities. In addition, we implement a precise and efficient algorithm for studying the exponential decay established in the first part of the paper numerically. Our simulations illustrate the efficacy of the proposed control design.

This is a section

Consider the problem

$$\begin{cases} i\,\partial_t\,\psi + \Delta\,\psi - |\psi|^2\,\psi + i\,\alpha\,b(x)\,(|\psi|^2+1)\psi = \phi\,\psi\,\chi_\omega \quad \text{in } \Omega\times(0,\infty) \\ \partial_{tt}\phi - \Delta\,\phi + u^5 + a(x)\,\partial_t\,\phi = |\psi|^2\,\chi_\omega \quad \text{in } \Omega\times(0,\infty) \\ \psi=\phi=0 \quad \text{in } \Gamma\times(0,\infty) \\ \psi(0)=\psi_0\in\, H_0^1(\Omega)\cap H^2(\Omega) \\ \phi(0)=\phi_0\in\, H_0^1(\Omega)\cap H^2(\Omega) \\ \phi'(0)=\phi_1\in\, H_0^1(\Omega)\cap H^2(\Omega) \end{cases} \tag{19}$$

where Ω is a bounded domain of $\mathbb{R}^N, N=3$ with smooth boundary Γ and ω is an open subset of Ω such that $\text{meas}(\omega)>0$ and satisfying the geometric control condition. In what follows, χ_ω represents the characteristic function, that is, $\chi=1$ in ω and $\chi=0$ in $\Omega\setminus\omega$. We consider $a,b\in L^\infty(\Omega)$ nonnegative functions such that

$$a(x)\geq a_0>0 \text{ a.e. in } \omega \quad \text{a.e. in} \quad b(x)\geq b_0>0 \text{ in } \omega \tag{20}$$

so that ω contains $\overline{\Gamma(x^0)}$, where

$$\Gamma(x^0)=\{x\in\Gamma:\,m(x)\,\cdot\,\nu(x)\geq 0\}\,. \tag{21}$$

We start with our first result regarding the well-posedness of the problem (19).

Theorem 14. *Given* $\{\psi_0,\phi_0,\phi_1\}\in\left[H_0^1(\Omega)\,\cap\,H^2(\Omega)\right]^2\,\times\,H_0^1(\Omega)$ *and assuming* (20) *holds and* $\alpha\geq\frac{5+16\,a_0}{2\,a_0\,b_0}$, *then, there exists a unique regular solution to the problem* (19) *such that*

$$\begin{aligned} &\psi\in\,L^\infty(0,\infty;H_0^1(\Omega)\,\cap\,H^2(\Omega)),\;\psi'\in\,L^\infty(0,\infty;L^2(\Omega)),\\ &\phi\in\,L^\infty(0,\infty;H_0^1(\Omega)\,\cap\,H^2(\Omega)),\;\phi'\in\,L^\infty(0,\infty;H_0^1(\Omega))\\ &\phi''\in\,L^\infty(0,\infty;L^2(\Omega))\,. \end{aligned}$$

The well-posedness of the problem (19) is obtained in a similar form to the works [1] and [9] through Faedo-Galerkin's method. Moreover, following the ideas employed in [1] and [9], the next result gives us the exponential stability results by employing a unique continuation property and observability inequalities:

Theorem 15 (Exponential Decay)**.** *Let $\{\psi, \phi\}$ be a regular solution to problem (19) given by Theorem (14) and assuming (20) holds and $\alpha \geq \frac{5+6\,a_0}{2\,a_0\,b_0}$. Then, for any $L > 0$, there are $C = C(L) > 0$ and $\gamma = \gamma(L)$ such that the following exponential decay holds*

$$E(t) \leq Ce^{-\gamma t}E(0), \text{ for all } t \geq 0$$

provided the initial data are taken in bounded sets of $\mathcal{H} := [H_0^1(\Omega) \cap H^2(\Omega)]^2 \times H_0^1(\Omega)$.

With regard to numerical experiments, the authors have planned to adapt some conservative numerical schemes to replicate the demonstrated stabilization result computationally. It is very important that the schemes are conservative since it would ensure that the dissipation of the solution is only due to the damping effects (if present) and not due to numerical or computational effects. For the Schrödinger case, we will consider what has already been done in [5, 6, 7]. For the Klein-Gordon case, we will use a Newmark scheme proposed in [8] for the time variable and finite differences in space.

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